Invited Lecture

(Re)Assessing Mathematics Education in The Digital Age

Alison Clark-Wilson[1]

ABSTRACT Digital technologies have been evident in the field of mathematics education since the late 1970s, a time of great optimism and enthusiasm for how emerging technologies would impact on school mathematics as a subject — and how mathematics would be taught and learned. Some fifty years on, whilst the pace of educational technology design accelerates, the parallel global transformation of school mathematics curricular — and associated high stakes assessment systems — lag noticeably behind. Within the mathematics education research field, there is general agreement about the barriers to systemic change: an underestimation of the professional needs of the teaching workforce; insufficient and inequitable access to suitable technologies; an unrealistic or ill-defined vision for students' digitally-enhanced mathematics learning experiences; challenges in the design and 'at-scale' uses of (mathematical) technologies in classrooms and its role within high-stakes assessments (Clark-Wilson, Robutti, & Thomas, 2020; Hoyles, 2018). The (re)emergence of computer programming, which was commonplace in UK mathematics classrooms of the 1980s has prompted some rethinking but, to date there are no widely accepted definitions of what a student's school mathematics educational experience in the digital age should comprise. The coronavirus pandemic prompted a global upskilling of students', parents' and teachers' digital skills within all phases of education and put technology, in its most general sense, on the map. In this invited lecture, I will offer a vision for how students' experiences of learning school mathematics *with and through* (mathematical) technologies might be reconceived. Alongside this, how the parallel assessment processes might be designed to enable a more student-centric approach that takes account of multiple sources of evidence. Most crucial to this is the role of teachers, whose expertise is more vital than ever as they support students to actively engage with substantive *dynamic* mathematical tools that make core mathematical ideas more tangible. The lecture concludes by highlighting how a deeper understanding of the theoretical construct of the 'hiccup' (Clark-Wilson, 2010; Clark-Wilson and Noss, 2015) might underpin wider understanding of the process of teachers' classroom-based learning concerning the adoption of mathematical technologies towards this vision.

Keywords: Dynamic mathematical technology; Hiccup; Cornerstone Math; Landmark activity; Mathematics teacher professional learning.

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1. Introduction

I am a former school mathematics teacher and a mathematics teacher educator who now works as a researcher at UCL Knowledge Lab within the IOE Faculty of Education and Society. I begin by thanking the ICME-14 Committee for inviting me to give this lecture.

In my lecture, I stand on the shoulders of the giants in our field to argue why, in the context of the fourth industrial revolution and its accelerating technological advancements, alongside the huge challenges that humanity faces across the globe, there is an urgent need to reassess what mathematics is taught in school — and how it is taught. In parallel, the critical examination of the nature and focus of associated high stakes assessments is necessary to ensure that we test the mathematical content and processes that we value. Furthermore, the education sector's interest in technology, which has been fuelled by the need for remote and hybrid teaching of mathematics during the 2020-21 pandemic period, has many now thinking seriously about the role that technology might play in the future of education around the globe. However, any changes to the mathematics curriculum, and its high stakes assessments will require the use of innovative and dynamic mathematical tools. The research field has documented and established over the last 20 years just how complex, and demanding it is for teachers to adopt and adapt the more epistemic digital tools into classroom practice. I'll expand on these challenges later in my lecture. My lecture includes many video images of dynamic mathematical tools, which can be accessed via the hyperlinks that are provided in the footnotes.

My lecture will be structured as follows. I'll begin with some personal reflections on how I began to use technology, initially as a learner of mathematics in the late 1970s and later as a teacher. I'll then move on to present my argument for why curriculum and assessment reform in mathematics is so urgently needed. This will be followed by an outline of the challenges relating to scaling and sustaining the integration of more dynamic mathematical tools, for which I will use the Cornerstone Math project from the UK as an example. My lecture will conclude with a focus on some key theoretical components of teachers' professional learning experiences and trajectories, and in particular, my own contribution; the construct of the lesson 'hiccup'.

2. A Personal Reflection on Learning and Teaching Mathematics with and through Technology

In my own mathematics education, I was a member of the very first cohort of school students in England who were permitted to use a digital calculating device — a calculator — in the high stakes' pre-university examinations (the Advanced level, or A-level). My newly acquired calculator replaced the need for me to use a paper booklet of tables of logarithms, trigonometric ratios and exponentials that had been in the mathematical toolkit of teachers and learners for the preceding 100 years.

Interestingly, my 1976 model Texas Instruments Calculator (Fig. 1) was marketed originally as an “electronic slide rule calculator”, which suggests that the calculator was the digital disruptor of its day, as it was positioned to replace the established tool of choice for many engineers and mathematicians, the slide rule.

Fig. 1. The Texas Instruments Electronic Slide Rule Calculator (1976)

As a school student, I fully embraced this calculator. I read its manual, I worked out what every button did, and I was genuinely interested in how it worked. How was it calculating square roots? How was it doing it so quickly? And I was particularly interested in the memory store and memory recall functionalities, finding all sorts of ways to help myself to be able to solve the problems at hand - to take advantage of its affordances. My A-level mathematics teacher, however, did not share the same enthusiasm. She saw my possession of a calculator as a disruption. I was one of only a handful of students in the class who were fortunate enough to own one, and my teacher was not at all curious about how this new technology would impact her teaching, and her students’ learning. Consequently, she left us to work out how to integrate the tool into our own learning experiences.

Ten years later I found myself beginning my professional life as a secondary school mathematics teacher, and the technologies had evolved to include: the LOGO programming language; spreadsheet and graphing softwares; and portable graphing calculators. These technologies were all being explored in the schools in which my teacher training placements took place. I observed many teachers using these technologies, and slowly began to plan and teach lessons that integrated technology. In the early 1990s, even though there were no school computer networks, data projectors or interactive whiteboards, we found ways to exploit the resources that we had. In this period, my personal curiosity and creativity was fuelled by these new mathematical tools. At first, I found myself re-examining my own mathematical knowledge and associated understandings, much of which I had acquired by using a very different toolkit. I became inquisitive about my teaching, and I began to develop as an action researcher. I was beginning to question how the affordances of these new tools were going to change not just the way I could teach, but perhaps the order in which I would approach the teaching of topics, and the ways in which I would think about student learning.

3. The Need for a (Re)Assessment of Mathematics Education

As I write this text in 2021, it is highly apparent that the speed, connectivity, interoperability, representational infrastructures and automation of the digital technologies that are now available demand us to question every aspect of the way in which we design, teach, assess and evaluate mathematics curricular around the world.

So let me present my argument. The vision offered in the OECD's manifesto *The Future of Education and Skills: Education 2030* highlights the need to prepare young people for lives and futures that will require them to be more adaptable, collaborative, critical, self-directed and resourceful than any previous generation (Organisation for Economic Co-operation and Development, 2018). But how do we prepare young people to solve problems for which there is no known, accepted, or even correct solution? One thing we do know is that mathematical ways of thinking will be a critical component. Secondly, the available technologies have moved on at great pace around us. We already have the technologies that enable rapid, accurate and a more natural mathematical user experience that will make some standard written algorithms redundant, or at least less important. In this context, how do we collectively *re-vision* the mathematics curriculum, and its assessment, to retain the integrity of the subject and its relevance to humanity? This may mean shaking off the shackles of international comparisons and the competitiveness of the international *race to the top*.

Let me start by highlighting some freely available mathematical tools that are shaking some of our foundations. Figure 2 shows a very typical looking question from a timed written examination that is designed to be answered only using paper and pencil tools — in this case from a high-stakes examination for 16-year-olds in England.

Work out the value of $\frac{3^7 \times 3^{-2}}{3^3}$

Fig. 2. A typical high-stakes examination question from England

Using a web-based tool, such as *Microsoft Math Solver* (Microsoft, 2019), I can scan the question with my iPad camera (Fig. 3) and solve the problem in just a few seconds (Fig. 4).

I can also take a more advanced typical question, for example to solve the pair of simultaneous linear equations, $5x + y = 21$ and $x - 3y = 9$. Again, a simple text scan (Fig. 5) can provide the numerical solutions in a range of different number representations. It also offers a choice of solution steps and the graphical representation (Fig. 6).

Handwriting recognition has also advanced greatly so, whereas previous computer algebra systems required us to learn and teach syntax - the specific language to talk to a computer — we can now use our more natural handwritten mathematical notations to the same output (Fig. 7).

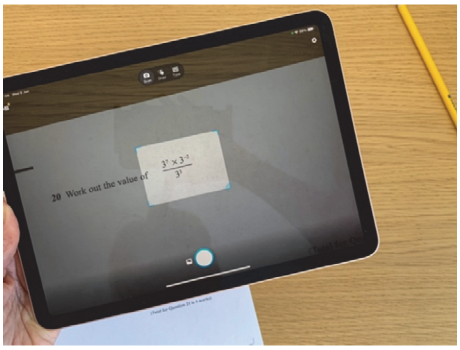

Fig. 3. Scanning the text

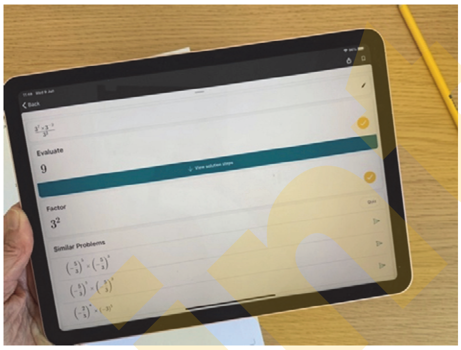

Fig. 4. The solution

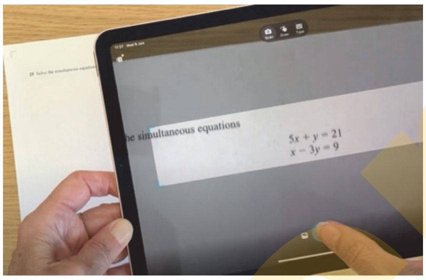

Fig. 5. Scanning the question text[2]

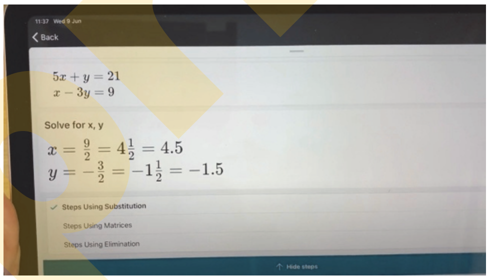

Fig. 6. The auto-generated response that gives multiple equivalent solutions as both products and processes

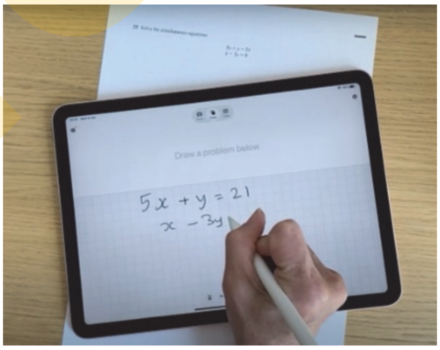

Fig. 7. Handwriting the problem[3]

[2]Video: Scanning and solving the problem. https://drive.google.com/file/d/1qp60FdT9eD8X5wbLS12XkjWOqBZsPKnH/view?usp=sharing
[3]Video: Handwriting and solving the problem. https://drive.google.com/file/d/1diP8xIK-dV52CF1Gru8Jb-8H1ZEgnN72/view?usp=sharing

Now, whilst *Microsoft Math Solver* offers a more transactional "input-output" environment, other mathematical technologies such as *Math Whiteboard* (Fluidity Software, 2019) enable handwritten mathematics *and* embed functionality that affords more natural explorations of mathematics — and perhaps offer opportunities for more pedagogical uses of such tools (Fig. 8). I urge you to view the video clip for this example. Tools such as *Math Whiteboard* also offer real-time cloud-based collaboration opportunities. We can be working on the same problems remotely and collectively.

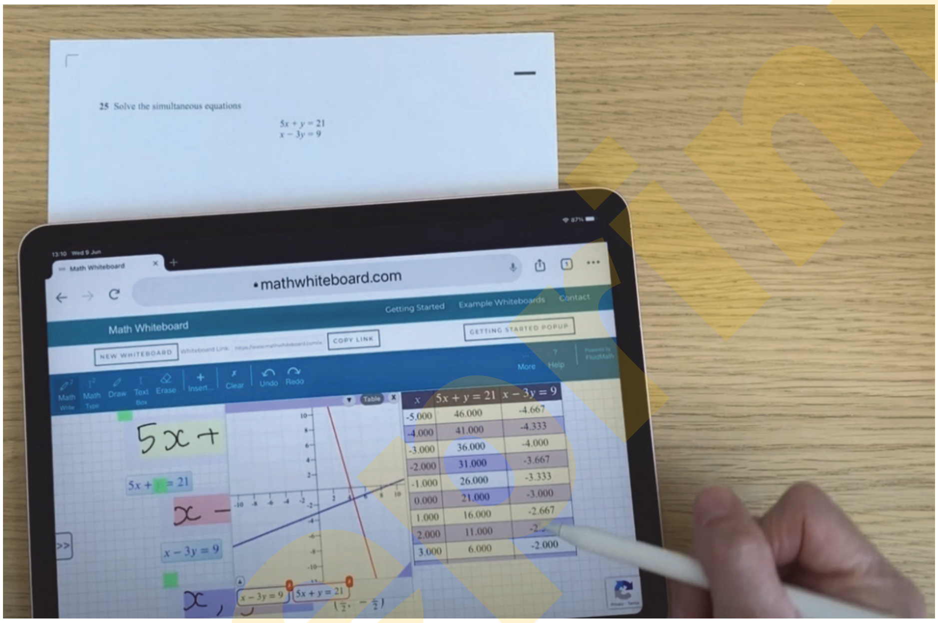

Fig. 8. Integrating handwritten and digital representations[4]

The examples above align very closely with the type of mathematics that is perceived to be the end goal for the teaching and learning mathematics for the majority of school students in many countries around the world. However, the availability and ease of use of the technologies shown above, may be seen to undermine the mathematics taught in school and be perceived by the community as facilitating *cheating*. An alternative perspective embraces such efficient and accurate tools in the same way that the calculator has now completely replaced paper and pencil methods for the calculation of square or cube roots. Hence, the mathematical work of the classroom becomes less about finding the answers to classical problems using drilled techniques, and more about the range, diversity, correctness, aesthetics and inherent mathematical beauty of different approaches to solving more contextualised problems that have wider relevance to society.

[4] Video: Integrating handwritten and digital representations.
https://drive.google.com/file/d/1fUYulU-gW3qY4jrvkKzbRRF6H8EW8XRH/view?usp=sharing

4. Towards a More Exploratory Approach to the Use of Digital Technologies in School Mathematics

So how do we move towards an integration of digital technology for mathematics that enables a more exploratory and inquiry-based approach for our learners? It is widely accepted that there are four main drivers of change within education systems that impact the way in which technology is taken up in mathematics classrooms, which are:

(1) *School mathematics curricula*, particularly where the jurisdiction has also mandated national/regional curricula, textbooks and (digital) resources, pedagogical approaches, etc.

(2) *Assessment processes*, which range from high-stakes testing to more teacher- or student-centric classroom or school-based approaches.

(3) *Support for teachers' lifelong professional learning*, to use digital technologies which, in the case of mathematics teachers, includes the more epistemic mathematical tools, which by their nature both *embed and represent* mathematics. The pace of technological developments mean there will always be a need to teachers to rethink and adapt such tools for classroom contexts.

(4) *National education strategies*, which attend to the accessibility and inclusivity of teachers' and students' uses of technology in all its forms. The related policies need to be continually updated such that the technology serve the needs of education rather than drive its use.

What follows is a case example of a country-wide project that began in 2013 to support more exploratory approaches to mathematics using the *Cornerstone Maths* digital mathematics curriculum units in England (UCL Institute of Education, 2017).

A case example: Cornerstone Math in England

The *Cornerstone Math* project was a multi-year collaboration between Stanford Research International (SRI) in the United States and colleagues at UCL Institute of Education, led by Richard Noss and Celia Hoyles. The project aimed to build on a series of earlier research projects that had developed dynamic mathematical technologies and had generated good evidence of improved students' learning outcomes. However, in each case, the digital resources required further research and development to enable them to be accessed, and used more easily, by teacher and (lower secondary age) students. Such research would aim to facilitate their use to be scaled into hundreds of schools in England (Hoyles et al., 2013).

Each of the *Cornerstone Math* curriculum units (Fig. 9) focused on an area of mathematics that was known to be *hard to teach* in lower secondary mathematics: early algebra and the notion of variable; linear functions; and geometric symmetry (to include trigonometric ratios).

This case example focuses on the unit on Linear Functions unit (*Designing Mobile Games*), the design of which was built on the very solid foundation of Jim Kaput and colleagues' work in the US. This unit embraces Kaput's seminal vision for the use of multi-representational technologies within context-based problem-solving tasks that aimed to democratize access to school mathematics for all learners (Hegedus and Lesh, 2008; Hegedus and Roschelle, 2013; Kaput, 1989, 2001).

Fig. 9. The *Cornerstone Maths* Curriculum Units

Each *Cornerstone Maths* unit embed the internet-based digital resources alongside *paper and pencil* task books for students. Alongside, teachers are provided with extensive guidance and professional support, which can take place both within and away from their schools. The *Cornerstone Maths* resources were developed over a period of 5 years using design-based research methodologies. Each iteration of the designs enabled both the technology (and the materials that were developed alongside) to evolve, and also for the research lens to be directed first towards student learning, then teacher learning, then scaling within England (and now in Taiwan and Indonesia). All *Cornerstone Maths* curriculum units are framed by three theoretical ideas:

(1) *Transformative technology*, "computational tools through which students and teachers (re-)express their mathematical understandings" (Clark-Wilson et al., 2015; Hoyles and Noss, 2003).

(2) *Scaling technology use in STEM education*: The processes and products that bring innovative technologies to most mathematics classrooms on a national level (as elaborated by Hung et al., 2010).

(3) *The "landmark activity"*: One that is indicative of a rethinking of the mathematics or an extension of previously held ideas. It is our assumption that disruptive but carefully designed technologies lead to a cognitive breakdown, or a "situation of non-obviousness" (Winograd and Flores, 1986,

p. 165). We devised the landmark activity as a methodological tool that enabled us to maintain focus when working with large groups of teachers and many hours of curriculum materials.

Now, I introduce you to one of the landmark activities from the curriculum unit on linear functions (Fig. 10), which you may have seen before, as it has been extensively researched, and features in many of our research group's publications and presentations! If not, I strongly encourage you to view the hyperlinked video to appreciate the dynamic nature of the multi representational environment.

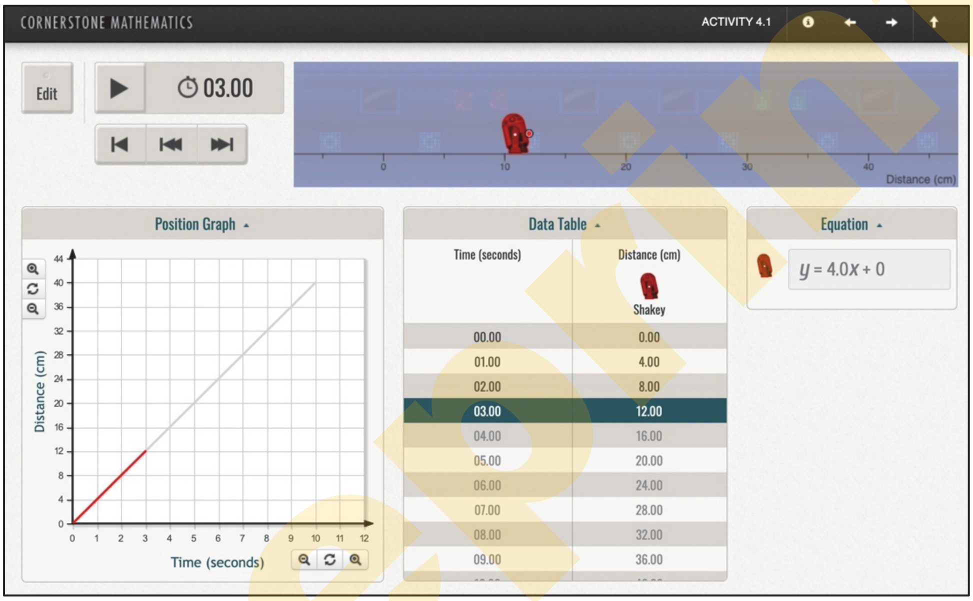

Fig. 10. Landmark activity: Shakey the Robot[5]

As you watch the animation, take time to look at the various representations on the screen. As Shakey begins to move, let your eyes wander around the screen. In a pedagogical setting I would encourage learners (which includes teachers in professional learning contexts) to carefully observe the different representations: a graph pane, a table of values, an equation, a character positioned on a number line and some control buttons that enable the animation to be played, stepped through (forward and back), stopped and rewound. As the character moves, you might also notice representations that are changing such as, the colour filling on the graph or some values being highlighted in the table. Did you also notice that some representations did not change? For example, the all-important invariant equation, which tends to go unnoticed by learners as they encounter the environment for the first time. The affordances of this environment enable some key mathematical ideas to be connected. For example,

[5] Video: Landmark Activity — Shakey the Robot.
https://drive.google.com/file/d/18o2hFuzjmsPMG8aKOBSsIeE0jFpgQxdC/view?usp=sharing

stepping through the animation second by second allows us to focus on how the characters motion is being represented in the different environments.

Selecting the *Edit* button unlocks this initial scenario, by revealing a number of editable objects, which take time to make sense of in the professional learning context. A number of objects, which we call "hotspots" can change the model, and each in a different way. A deep understanding of how varying the position of each hotspot impacts the resulting new mathematical scenario is crucial for teachers as they begin to consider (and support) the classroom discussions they (might) have with learners. Although the teacher guide and pupil workbooks provide many prompts for such discussions, our research findings concluded the importance of teachers' classroom discourse, underpinned by a level of tool fluency that fully exploits its dynamic and epistemic nature (Clark-Wilson and Hoyles, 2017; Simsek, 2020).

The above activity is the fourth of a series of twelve. It is the activity where students *first* meet the algebraic representation of a function/equation. The previous activities focus on the relationships between the animation, graph and table of values. Our approach contrasts with the traditional way that the graphing of functions is introduced, where students are expected to learn to use the function to produce the table of paired values, followed by plotting these values as coordinate points to create the graph. The Cornerstone Maths technology enables this topic area to be approached in reverse. Students are given time to make sense of the animation, the position graph and the table of values prior to meeting the algebraic syntax of the function. The language of the character (i.e., describing how Shakey is moving) supports students to be able to explain and articulate their models and explore different scenarios (i.e., making Shakey disappear or move backwards). Hence the equation is first experienced as a context-specific syntactic representation that generalises the situation at hand.

The big mathematical ideas that are addressed within the complete unit are: the coordination of algebraic, graphical, and tabular representations; the use of speed as a context to introduce rates of change; explorations of $y = mx + c$ as a model of constant velocity motion (the meaning of m and c in the motion context); and the definition of velocity as speed with direction; and the concept of average velocity. Alongside, the design principles for the technology include: offering dynamic simulations and linking between representations; the facility to 'drive' the simulation from the graph or the function; and the ability to show or hide representations, as appropriate.

I now move to outline some of the more effective teaching practices that were revealed by our many observations of the previously described landmark activity. These include:

- Emphasising strongly the need for pupils to make sense of the 'hotspots' that facilitated the graph to be edited.
- Emphasising the multiplicative/additive relationships in the table to justify the meaning of the equation.

- Highlighting the invisible variant m within the table for equations with a non-zero value of c.
- Paying attention to the graph's axes and promoting discussion of the effect of changing the scale.
- Gathering back the students' multiple responses - as particular cases - in order to support the overarching generalisation that the greater the value of m, the faster that Shakey will move.
- Extending Shakey's journey time such that his final position could not be read from the graph nor the table — to provoke pupils to use the equation to calculate its position after a given time and thus highlighting the power of the mathematical equation as a generalisation.

Although these are all promising practices, they were collectively observed across a small number of classrooms by fewer than ten teachers (Clark-Wilson and Hoyles, 2017). The vast majority of teachers *did not interact with the technology at all* when they were discussing the students' task outcomes in either small groups or when leading whole-class plenaries. We conclude that the development of classroom teaching that embeds the use of dynamic, epistemic digital tools such as *Cornerstone Maths* requires teachers to have professional learning time alongside, and opportunities to reflect individually and with others, over years, rather than weeks or months (Clark-Wilson and Hoyles, 2019).

5. How do Teaching Practices with Dynamic Digital Technologies Evolve?

Professional programs, training events, collaborative projects, and self-directed learning, are all fundamental for teachers to come to know new technologies that might support the teaching and learning of mathematics. Ideally, driven by teachers' own curiosities, and supported through collaboration with others, new teaching ideas emerge, which are sometimes shared within different communities. The nature of the technology that is selected, brings different challenges — adopting a generic online quiz technology is a very different technology to a more epistemic dynamic geometry or graphing application. Similarly, the type of technologies that are suited to younger learners differ greatly than those for older learners. I focus what follows on the more epistemic mathematical technologies within the context of mathematics teachers in England within the secondary phase (11–16 years).

My own doctoral research sought to understand and theorise the way that teachers' learning evolved through their classroom use of a new technology (the TI-Nspire handheld and software). The teachers' common purpose for the use of the technology was to support their learners to develop mathematical generalisations through teacher-designed tasks that promoted explorations of related variant and invariant properties. The study was framed by Verillon and Rabardel's (1995) *instrumental approach*, which had been elaborated for mathematics education by the seminal work of Guin and Trouche (1999). Hence the processes of *instrumentation* (coming to know the tool and its affordances) alongside *instrumentalization* (learning to exploit the tool for a mathematical/pedagogical purpose) would underpin my analysis of the teachers' actions in the classroom.

My research concludes the theoretical construct of the "lesson hiccup", which I define as "the perturbations experienced by the teachers during the lesson, triggered by the use of the technology that illuminated discontinuities in their knowledge." (Clark-Wilson, 2010, p. 138). Hiccups are phenomena that are: "highly observable" and "cause the teacher to hesitate or pause" (ibid). The analysis of 14 lessons taught by two teachers over a period of nine months resulted in 66 hiccups, which were each categorized as one of seven hiccup types, which are described in Tab. 1.

Tab. 1. The seven types of lesson hiccups experienced by teachers (Clark-Wilson, 2010)

Hiccup type	Relating to the:
Task design	Choice of initial example. Sequencing of examples. Labelling of objects. Pedagogical approach.
Interpretations of the mathematical generalisation	Specific ⇔ general case. Range of permissible responses. Failure to notice.
Unanticipated student responses	Students' prior understanding. Student's misinterpretation of activity objectives. Students' own approaches.
Student perturbations	Unanticipated outputs from the digital tool. Doubting the authority of the digital tool
Students' instrumentation issues	Making inputs to the digital tool Grabbing and dragging dynamic objects Organising on-screen work Navigating between representations Accidental deletion
Teacher's instrumentation issues	Forgotten (or not yet learned) techniques
Unavoidable technical issues that are out of the teacher's control	Classroom network failure Technology failure

Appreciating the nature of the hiccups that teachers experience in their classrooms for particular mathematical technologies could be key to designing research-informed professional learning opportunities for teachers, alongside supporting the communities' reflective processes. By definition, a hiccup only exists if the teacher has experienced the technology-triggered perturbation and it is my hypothesis that lessons that involve dynamic mathematical technologies provoke many hiccups for most teachers — particularly during the early lessons. Professional learning models such as Lesson Study, and its many cultural and contextual adaptations, offer promising scenarios for the hiccup construct to be adopted as a lens to support teachers' collective lesson design, classroom enactment and associated reflection processes.

6. Rethinking the Assessment of School Mathematics

I would like to return to the topic of assessment. If we have redesigned mathematics curricular in which students' technology-enhanced classroom activities require them to produce mathematical work that embeds the use of dynamic mathematical tools, how does that change what and how we might assess? I propose that, rather than grappling with ways to manage the integration of digital technologies to synchronous exam room settings (which requires considerable resources), we look to how digital tools might altogether transform the way that we conceive the body of knowledge that comprises school mathematics.

One such digital tool is the Cambridge Maths Framework[6], a conceptual mapping tool which has been over 6 years in development. It comprises a searchable network of key mathematical ideas and the relationships between them, that spans the domain of school mathematics and is informed by over 1400 research sources and expert consultation. Evident within the framework, is the key construct of "waypoints", the mathematical ideas that have emerged as being multiply connected and/or crucial for the progression of later ideas (Fig. 11).

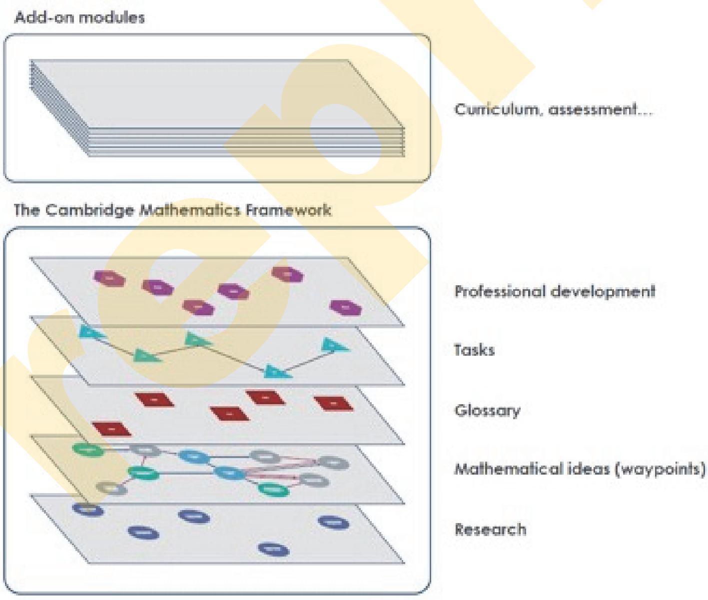

Fig. 11. Connected layers within the CM Framework and external add-on modules (Koch et al., 2021, p. 128)

The dynamic digital network graph format of the CMF allows external content from curricula or resources to be manually "mapped on" to CMF content so that it can be analysed according to the conceptual orderings, interdependencies and justifications

[6] https://www.cambridgemaths.org/

from the research literature represented in the CMF. There are two potential applications of such a tool for the redesign of assessment goals and associated processes.

(1) The development of more naturalistic assessments of waypoints that can be taken by students “when ready” and involve the use of digital technological tools, where appropriate. Students’ assessment outcomes could be recorded on their personal and portable record of mathematical achievement that could have a more universal application and understanding within global education contexts. Such an approach might lead to a (re)visioning of teachers’ professional roles as assessors of their students’ mathematical understandings.

(2) Designers of digital technologies that already collect or collate students’ assessment outcomes relating to waypoints might develop interoperability with locally contextualized versions of the framework to enable a more real-time approach to assessment. This approach might reduce the status of high-stakes examinations to account for alternative forms of assessment that support education systems to evolve.

I do not envisage the CMF as a tool to support the micro-assessment of all aspects of a learners’ mathematical journey. Whilst it helps us to think about the smaller processes that might underpin conceptual development, it is more helpful as a tool to help us determine what to assess, in what sequence and how.

Returning to the examination question that assessed simultaneous equations that I highlighted earlier (Fig. 5), how might this concept be assessed within an environment where students have access to dynamic mathematical tools? Fig. 12 shows another

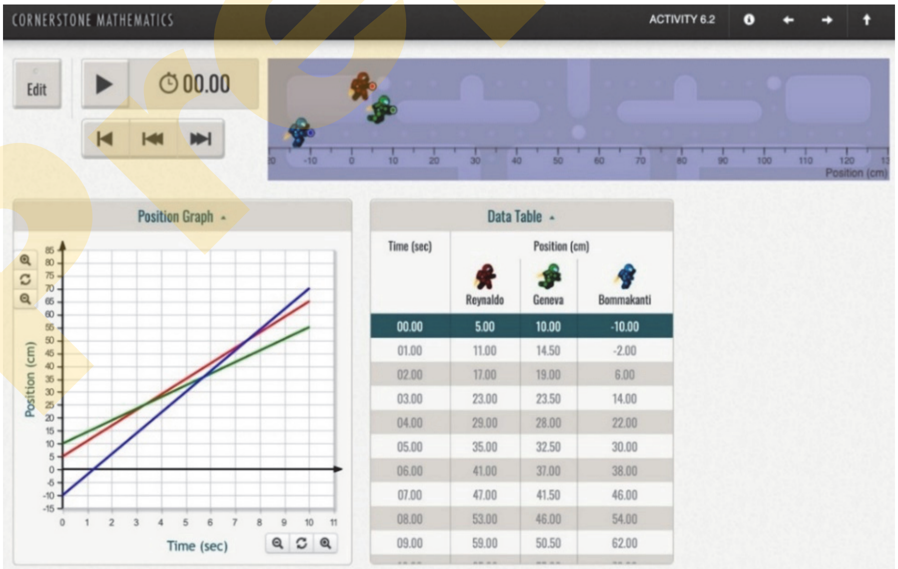

Fig. 12. Modelling simultaneous equations

scenario from the *Cornerstone Maths* curriculum, which includes three characters, each of which are designed to move at different speeds.

The scenario provides multiple opportunities for students to create and justify models that might evidence their understanding of linear functions in general, and the notion of simultaneous functions, in particular. Again, the context of the scenario (characters in an animation game that move according to some criteria), offers a familiar environment to mathematicise *coinciding* or *overtaking*. This increases the accessibility of the mathematics, whilst also offering contextual language to support both explanations and applications of the associated knowledge.

7. Concluding comments

To conclude, I'd like to introduce two ideas to stimulate the community to think about the nature of learners' mathematical experiences in a world where they have increasing access to a blend of digital and non-digital resources in, and away from the classroom. I'll address this first at a macro level, by considering the overall picture for any particular learner.

My "healthy learning plate" (Fig. 13) is elaborated for school mathematics from Laurillard's pedagogic theory, "the Conversational Framework", which was designed to consider digital transformation of students' learning experiences within the context of higher education (Laurillard, 1993, 2002).

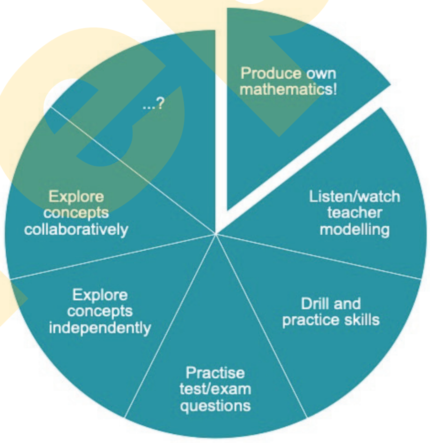

Fig. 13. A digitally enhanced school maths "healthy learning plate" — Adapted from Laurillard's pedagogic theory — the "Conversational Framework" (1993, 2002)

The plate shoes the main forms of learning activity that a learner might experience, which I offer as a tool to reflect on the balance of these experiences for learners at different stages of their mathematical development. In doing so, we might consider:

- Which of these experiences might be added, or taken away?

- What proportions of the different experiences are desirable?
- Which of these experiences might take place in, or away from the classroom?
- How do we ensure that, as the stakes get higher, and the balance of their experiences might shift, learners do not lose out on the most mathematically nutritional elements?
- How balanced are our current approaches?

My second idea is focusses attending more carefully to what we really mean when we talk about *doing mathematics*. In our field, this idea is very widely and broadly addressed. If we seek to collate lists of terms that capture human experiences when doing mathematics, we arrive at many lists that have traditionally been associated with different mathematical content domains. However, as the mathematical digital tools merge the boundaries between these domains, it is time to take a more holistic approach. Fig. 14 includes my own list on which I have highlight those processes that were exemplified in the Cornerstone Maths tasks earlier.

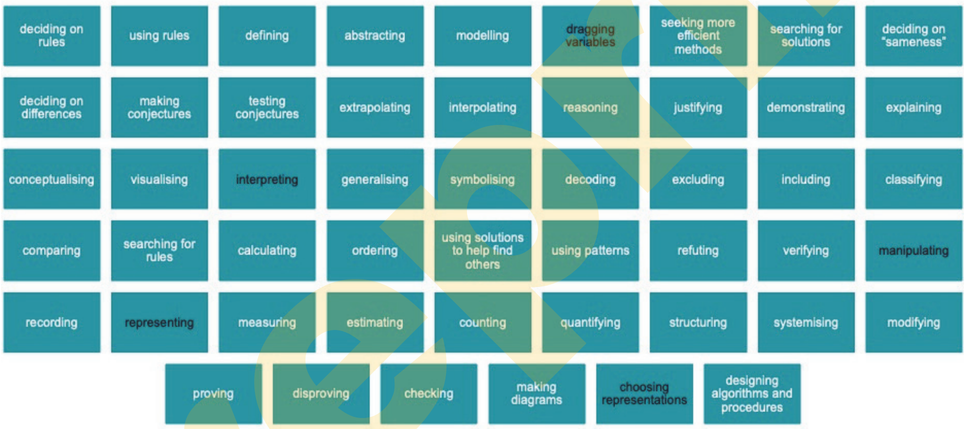

Fig. 14. A digitally enhanced school maths "healthy learning plate" — Adapted from Laurillard's pedagogic theory — the "Conversational Framework" (1993, 2002)

It is these processes that might be:

- noted and reflected upon by the learners themselves.
- observed by teachers and parents.
- captured and logged by the digital tools to inform the design of more appropriate assessments.
- used by all to enrich task and resource designs (including textbooks).

In my view the increasing access to, and use of technology within mathematics education around the world should prompt us to (re)assess how we can move towards mathematical experiences for all learners that promote rich mathematical experiences. Alongside, we must take advantage of the affordances of the digital and non-digital resources to design more authentic assessment approaches that align with nurturing all children to develop confidence and competence in mathematics.

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